The numerical solution presented here is capable of describing non-Newtonian jet flow under realistic conditions (finite width orifice, any type of initial profiles) and has certain advantages over the similarity solution. However, in many instances it might be easier to use the similarity solution. It is possible to improve the results of the similarity analysis by defining a virtual origin at the jet, that is, shifting the origin of jet to fit experimental data or the present numerical results. Such an improvement has been suggested by Pai (1972), for Newtonian jets.

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NOTATION

$b_{1/2}$ = half-jet width, that is, locus of points for which $U = \frac{1}{2} U_m$
D = orifice width
m = consistency index of the power law model
n = power law index
Re = Reynolds number, $D^n u_0^{2-n} \rho / m$
u = velocity in main flow direction x
u_0 = maximum velocity at the orifice
U = dimensionless velocity, $= u/u_0$
U_i = velocity profile at the orifice (initial)
U_m = velocity at the jet midplane
v = velocity in the y-direction
V = dimensionless velocity, $= vRe/u_0$
x = coordinate in the main flow direction
X = dimensionless coordinate, x/DRe
y = coordinate perpendicular to main flow direction
Y = dimensionless coordinate, y/D
ΔX = X-direction stepsize
ΔY = Y-direction stepsize
ρ = density
τ = shear stress

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Some Notes on the Temperature Response of Packed Beds

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Design and control problems in connection with various kinds of plants in which a bed of solid material is transported by a grid or grate while being processed by a cross flow of fluid have revived the interest in the responses of packed or porous beds (Voskamp et al., 1972). Some of these problems have already been discussed as early as in 1926 in connection with the heat recuperator (Anzelius, 1926) see also the survey by Jakob (1957), but changed conditions make it worthwhile to reconsider the problems. Kohlmayr (1968) did this a few years ago, presenting a new pair of analytical solutions claiming that they lend themselves to more efficient computation than Schumann's analytical solution, and that it is no longer necessary to resort to finite-difference methods when computing theoretical response functions.

The present paper submits another analytical solution based on a finite-difference approach of which the solution presented by Kohlmayr is a special case. Ironically, our solution was developed in 1968, but only recently did we discover that in digital computations it is more efficient than any other solution we have tried, except when an unusually high accuracy is desired.

BASIC EQUATIONS

The following set of equations describe the bed of spherical particles, heated or cooled by a fluid:

$$\frac{1}{\kappa}\,\frac{\partial T_s}{\partial t} = \frac{\partial^2 T_s}{\partial r^2} + \frac{2}{r}\,\frac{\partial T_s}{\partial r}, \quad \kappa = \lambda_s/(\rho_s \gamma_s) \tag{1}$$

$$T_s\{r, 0\} = \text{given function of } r. \tag{2}$$

For ease of discussion, the bed is divided into N horizontal layers, N being chosen so large that vertical temperature differences within any layer are negligible.

Neglecting the heat capacity of the gas holdup, the heat balance of the gas for one layer can be written as

$$F_g \gamma_g T_{g,n-1} + Q_{s,n} = F_g \gamma_g T_{g,n} \tag{3}$$

where γ_g represents the specific heat of the gas (assumed constant). Further

$$\left.\frac{\partial T_{s,n}}{\partial r}\right|_{r=R} = -\,[T_{s,n}\{R, t\} - T_{a,n}\{t\}]\,\frac{U}{\lambda_s} \tag{4}$$

$$Q_{s,n} = F_g\gamma_g\,(T_{g,n} - T_{g,n-1}) = [T_{s,n}\{R, t\} - T_{a,n}\{t\}]\,\frac{UA}{N} \tag{5}$$

where $A/N = \sigma 4\pi R^2$ represents the surface area of the particles in the layer with

$$T_{a,n} = \frac{1}{2}\,(T_{g,n-1} + T_{g,n}) \tag{6}$$

SOLUTION

Spheres Having Internal Temperature Gradients

Substitution of

$$u_n\{r, t\} = r \cdot T_{s,n}\{r, t\} \tag{7}$$

into (1), (4) and (5) yields, for layer n, respectively,

$$\frac{\partial u_n}{\partial t} = \kappa\,\frac{\partial^2 u_n}{\partial r^2} \tag{8}$$

$$\frac{1}{R}\,\frac{\partial u_n\{r, t\}}{\partial r}\bigg|_{r=R} = -\left[\frac{u_n\{R, t\}}{R} - T_{a,n}\{t\}\right]\frac{U}{\lambda_s} + \frac{u_n\{R, t\}}{R^2} \tag{9}$$

$$Q_{s,n} = F_g\gamma_g(T_{g,n} - T_{g,n-1}) = \left[\frac{u_n\{R, t\}}{R} - T_{a,n}\{t\}\right]\frac{UA}{N} \tag{10}$$

where $T_{a,n}$ is given by (6).

The Laplace transformation of (8) with respect to time (introducing q as the Laplace variable) yields a second-order ordinary differential equation in r which is easily solved, provided the initial solid temperature is independent of the radius:

$$T_{s,n}\{r, 0\} = T_{s,0}, \quad u_n\{r, 0\} = r \cdot T_{s,0} \tag{11}$$

Then the solution is

$$u_n\{r, q\} = \frac{\sinh\left(r\sqrt{\frac{q}{\kappa}}\right)}{\sinh\left(R\sqrt{\frac{q}{\kappa}}\right)}\left[u_n\{R, q\} - \frac{RT_{s,0}}{q}\right] + \frac{rT_{s,0}}{q} \tag{12}$$

Now $u_n\{R, q\}$ can be found by differentiation of (12) with respect to r and substitution into the transform of boundary condition (9), which yields

$$\frac{u_n\{R, q\}}{R} = \frac{1}{G\{q\}}\left(T_{a,n} + \frac{\lambda_s}{UR}\left[R\sqrt{\frac{q}{\kappa}}\,\mathrm{cotgh}\left(R\sqrt{\frac{q}{\kappa}}\right) - 1\right]\frac{T_{s,0}}{q}\right) \tag{13}$$

where

$$G\{q\} = 1 + \frac{\lambda_s}{UR}\left[R\sqrt{\frac{q}{\kappa}}\,\mathrm{cotgh}\left(R\sqrt{\frac{q}{\kappa}}\right) - 1\right] \tag{14}$$

$u_n\{r, q\}$ can be obtained by substitution of (14) into (13). λ_s may be replaced by $3\kappa\mu_{s,n}N/AR = 3\kappa\mu_s/AR$ where μ_s is the total heat capacity of the bed. Substitution into (13) and the transformed version of (10) gives

$$T_{g,n}\{q\} = \frac{1 + \tau_1\frac{3\kappa}{R^2}\left(R\sqrt{\frac{q}{\kappa}}\,\mathrm{cotgh}\left(R\sqrt{\frac{q}{\kappa}}\right) - 1\right)}{1 + \tau_2\frac{3\kappa}{R^2}\left(R\sqrt{\frac{q}{\kappa}}\,\mathrm{cotgh}\left(R\sqrt{\frac{q}{\kappa}}\right) - 1\right)}\left[T_{g,n-1}\{q\} - \frac{T_{s,0}}{q}\right] + \frac{T_{s,0}}{q} \tag{15}$$

with

$$\tau_1 = \mu_s\left(\frac{1}{UA} - \frac{1}{2F_g\gamma_gN}\right), \quad \tau_2 = \mu_s\left(\frac{1}{UA} + \frac{1}{2F_g\gamma_gN}\right) \tag{16}$$

The inverse Laplace transformation of (15) is simple if the gas inlet temperature is constant:

$$T_{g,n-1}\{q\} = T_{g,0}/q \tag{17}$$

Choosing $T_{s,0}$ as zero point of the temperature scale, the solution is given by

$$T_{g,n}\{t\} = T_{g,0} - \sum_{l=1}^{\infty} C_l \exp\{-p_l^2\,\kappa t/R^2\} \tag{18}$$

in which

$$C_l = \frac{2(1 - \tau_1/\tau_2)\,(\sin p_l/p_l - \cos p_l)}{1 - \cos p_l \cdot \sin p_l/p_l}\sin p_l/p_l \tag{19}$$

where p_l represents the roots of

$$p_l\,\mathrm{cotg}\,(p_l) - 1 + R^2/(3\kappa\tau_2) = 0 \tag{20}$$

Equation (18) gives the output gas temperature response of a layer if the input gas temperature is constant for $t > 0$. Differentiation with respect to time yields the impulse response of one layer and by means of convolution integrals the behavior of the complete bed could be calculated, but we found this to be an inefficient and inaccurate procedure. It is better to work out the expressions for $T_{g,n}$ in the Laplace domain by repeated application of (15), as illustrated later, and to obtain the desired responses by numerical inverse transformation.

Yet the response formulae (18) to (20) are useful because the incoming gas stream often has a constant temperature and the largest gradients occur in the first layer of the bed that comes into contact with it. The formulae can be used to check the internal temperature gradients in this layer. If these can be neglected, then the internal temperature gradients are negligible for all layers in the bed, provided it is homogeneous.

It is further interesting to note that the solution is governed by only three dimensionless groups, namely,

$$t/\tau_2, \quad \tau_1/\tau_2 \quad \text{and} \quad \kappa\tau_2/R^2 \tag{21}$$

One interesting possibility of exploiting the above results is to determine how these three groups affect the responses of the bed. Finally, by developing the transcendental functions into series, approximate solutions can be obtained. Such provide an opportunity to check whether the internal temperature gradients have to be accounted for or not.

Irregular Particles Without Internal Temperature Gradients

In this case the solid material of each layer has a uniform temperature $T_{s,n}\{t\}$. Taking lim $\lambda_s \to \infty$, $T_{g,n}\{q\}$ can be found as

$$T_{g,n}\{q\} = \frac{1 + q\tau_1}{1 + q\tau_2}\left[T_{g,n-1}\{q\} - \frac{T_{s,n,0}}{q}\right] + \frac{T_{s,n,0}}{q} \tag{22}$$

with τ_1 and τ_2 as defined in (16). It is interesting to note that this equation has exactly the same structure as (15), only the transfer function before the square bracket being different.

From (22) it follows that a bed having N layers, each with initial conditions $T_{s,n,0} = T_{s,0}$, the relation between input gas temperature $T_{g,\text{in}}\{q\}$ and output gas temperature $T_{g,N}\{q\}$ is given by

$$T_{g,N}\{q\} = \frac{1}{q}\left[\frac{1 + q\tau_1}{1 + q\tau_2}\right]^N [q\, T_{g,\text{in}}\{q\} - T_{s,0}] + T_{s,0} \tag{23}$$

If $T_{g,\text{in}}\{t\}$ is constant, then $q\, T_{g,\text{in}}\{q\} = T_{g,\text{in}}$ and the inverse temperature transformation of (23) can be found by applying Newton's binomial formula as follows:

$$\frac{v^N}{q} = \frac{1}{q}\left[\frac{1 + q\tau_1}{1 + q\tau_2}\right]^N = \frac{1}{q}\left[\frac{\tau_1}{\tau_2}\left(1 + \frac{1/\tau_1 - 1/\tau_2}{q + 1/\tau_2}\right)\right]^N$$
$$= \left[\frac{\tau_1}{\tau_2}\right]^N \left[\frac{1}{q} + \sum_{r=1}^{N} \binom{N}{r} \frac{1}{q} \frac{(1/\tau_1 - 1/\tau_2)^r}{(q + 1/\tau_2)^r}\right] \tag{24}$$

Using

$$\mathcal{L}^{-1}\left[\frac{1}{q}\,\frac{1}{(q + 1/\tau_2)^r}\right] = \tau_2{}^r\left\{1 - \sum_{n=0}^{r-1} \frac{(t/\tau_2)^n}{n!} e^{-t/\tau_2}\right\}$$
$$r = 1, 2, \ldots N, \tag{25}$$

it follows that

$$\mathcal{L}^{-1}\left[\frac{v^N}{q}\right] = \left[\frac{\tau_1}{\tau_2}\right]^N \left\{1 + \sum_{r=1}^{N} \binom{N}{r} \left[\frac{1}{\tau_1} - \frac{1}{\tau_2}\right]^r \tau_2{}^r + \right.$$
$$\left. - \sum_{r=1}^{N} \binom{N}{r} \left[\frac{1}{\tau_1} - \frac{1}{\tau_2}\right]^r \tau_2{}^r \sum_{n=0}^{r-1} \frac{(t/\tau_2)^n}{n!} e^{-t/\tau_2}\right\} \tag{26}$$

The first part of this expression can be simplified by another application of Newton's formula

$$1 + \sum_{r=1}^{N} \binom{N}{r} \left[\frac{\tau_2}{\tau_1} - 1\right]^r = \left[1 + \frac{\tau_2 - \tau_1}{\tau_1}\right]^N = \left[\frac{\tau_2}{\tau_1}\right]^N \tag{27}$$

which reduces (26) to

$$\mathcal{L}^{-1}\left[\frac{v^N}{q}\right] = 1 + g_N\{t\} \cdot e^{-t/\tau_2} \tag{28}$$

in which

$$g_N\{t\} = -\left[\frac{\tau_1}{\tau_2}\right]^N \sum_{r=1}^{N} \binom{N}{r} \left[\frac{\tau_2 - \tau_1}{\tau_1}\right]^r \sum_{n=0}^{r-1} \frac{(t/\tau_2)^n}{n!} \tag{29}$$

Thus the transform of (23) can be written as follows:

$$T_{g,N}\{t\} = [1 + g_N\{t\}\, e^{-t/\tau_2}]\, [T_{g,\text{in}} - T_{s,0}] + T_{s,0} \tag{30}$$

It is interesting to note that a different formula is obtained by expanding v^N/q as follows [compare Equation (24)]:

$$\frac{v^N}{q} = \frac{1}{q}\left[\frac{1 + q\tau_1}{1 + q\tau_2}\right]^N = \frac{1}{q}\left[1 + \frac{\tau_1 - \tau_2}{\tau_2}\,\frac{q}{q + 1/\tau_2}\right]^N =$$
$$= \frac{1}{q} + \sum_{r=1}^{N} \binom{N}{r} \frac{1}{q}\left[\frac{\tau_1 - \tau_2}{\tau_2}\,\frac{q}{q + 1/\tau_2}\right]^r \tag{24a}$$

we find in a similar fashion a different but equivalent expression for $g_N\{t\}$

$$g_N'\{t\} = \sum_{r=1}^{N} \binom{N}{r} \left[-\frac{\tau_2 - \tau_1}{\tau_2}\right]^r \sum_{n=0}^{r-1} \binom{r-1}{n} \cdot \frac{(-t/\tau_2)^n}{n!} \tag{29a}$$

If N approaches infinity, the functions $T_{g,N}$ approach the solutions of the partial differential equation given in the literature (Kohlmayr, 1968).

DISCRETISATION RULE

From (16) it follows that

$$\frac{\tau_1}{\tau_2} = \frac{1 - \beta/N}{1 + \beta/N}, \quad \beta = \frac{UA}{2F_g\gamma_g} \tag{31}$$

τ_1/τ_2 being determined by the number of layers N and a dimensionless parameter β. This parameter is proportional to the rate of heat transfer per degree temperature difference between solid and gas, divided by the rate of heat transport by the gas per degree temperature rise.

If the bed is made twice as high, only the heat-transfer area A, and hence also β, becomes twice as high. Therefore, β can be regarded as a (dimensionless) measure of the height of the bed. Formulated rather bluntly: if β is small, the dynamic behavior is governed mainly by the resistance to heat transfer between solid and gas and the number of layers is unimportant, while if β is large, the behavior depends primarily on the transport of heat by the gas, which implies that the vertical temperature gradient—and hence the number of layers—is of prime importance.

A tentative analysis revealed that as a rule of thumb N should be chosen larger than $1 + \beta$. Figure 1 shows the maximum error that can occur when this discretisation rule is applied. In this figure, the maximum deviation of the lumped from the exact model as regards the outlet gas temperature is given as a fraction of the initial temperature difference between gas and solid.

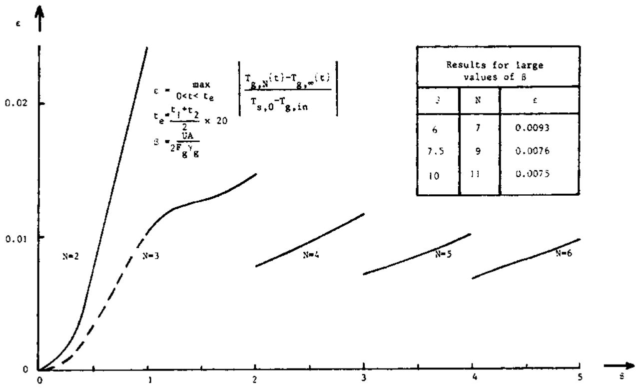

Fig. 1. The maximum error of the outlet gas temperature response of a lumped model as a function of the dimensionless bed height β and the number of layers N.

NOTATION

A = heat transfer area
C_l = coefficient defined by (19)
F_g = gas mass flow
$g_N\{t\}$ = time function defined by (29)
$G\{q\}$ = function of q, defined by (14)
$\mathcal{L}^{-1}$ = inverse Laplace transform
n = integer, indicating the layer concerned
N = total number of layers
p_l = the roots of equality (20)
q = transform domain of t
Q_s = heat flow to the gas
$Q_{s,n}$ = heat flow to the gas in layer n
r = distance from the centre of the sphere
R = radius of the spheres
t = time
$T_{a,n}$ = the average gas temperature in layer n
$T_{g,n}$, $T_{g,n-1}$ = the temperature of the gas leaving layer n and layer $n-1$, respectively
$T_{g,N}$ = the temperature of the gas at the outlet (of layer N)
$T_{g,0}$ = the constant inlet gas temperature
T_s = solid temperature
$T_{s,0}$ = initial solid temperature
$T_{s,n}$ = solid temperature of layer n
u_n = see Equation (7)
U = heat transfer coefficient
z = coordinate of height

Greek Letters

β = half the normalized height, see Equation (31)
γ_g, γ_s = specific heat of gas and solid material
κ = thermal diffusivity of the solid material
λ_s = thermal conductivity of the solid
μ_s = heat capacity of the solid
$\mu_{s,n}$ = heat capacity of the solid of layer n
ρ_s = specific density of the solid
σ = the number of particles in the layer
τ, τ_1, τ_2 = time constants

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Dropwise Condensation

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It has been recently reported by Lauro and Deronzier (1973) and Niezborala and Claudel (1973) that the fluorocarbon disulfide $(C_8F_{17}C_2H_4S)_2$ is an excellent long lived (500 hr.) drop promoter. Such a compound has been suggested by Bromley, Porter, and Read (1968). A sample of this especially made material was obtained for testing from Ugine Kuhlman through the courtesy of J. Huyghe.* A comparison test of this promoter was made with the two best promoters used by Wilkins and Bromley (1973): tetrakis octadecyl thio silane and n-octadecyl mercaptan.

EXPERIMENT

Equipment was assembled for life testing of drop promoters consisting of 4 vertical copper tubes 9.5-cm O.D. with 18 cm exposed to steam but in separate parallel chambers. The steam was prepared by boiling acidified and degassed sea water on a once-through basis. The copper tubes were air cooled for convenience, resulting in a low heat flux of about 5000 W/m^2 (1,600 B.t.u./hr. sq. ft.). The tubes were all initially cleaned with nitric acid, washed with distilled water, and immediately installed. Five cubic centimeters of a 1% solution of drop promoter in 2-ethyl hexanoic acid (warmed if necessary) were injected at the top of the tube with steam present and normal condensation occurring.

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Two sets of uninterrupted tests were run. In the first set one tube was left unpromoted. In the second set the tube with the fluorocarbon disulfide was interchanged with the octadecyl thio silane; as in the first test it was noted that venting did not appear to be identical for the four tubes. Also, the unpromoted tube was replaced with one using previously prepared (2 months) solution of thio silane.

The first set was run continuously for one month and the second for six weeks (1000 hr.).

RESULTS

Except for the unpromoted tube which was largely filmwise after a day, all tubes gave 100% excellent dropwise condensation at first.

The fluorocarbon disulfide was the first to show failure with 100% dropwise condensation lasting from 1 week (the first set) to 3 weeks (the second set). Filmwise condensation was essentially complete in 2 more weeks with considerable tube darkening evident.

The octadecyl mercaptan produced 100% dropwise condensation for 3 to 5 weeks but was 100% filmwise after 6 weeks with some tube darkening.

The tetrakis octadecyl thio silane gave 100% dropwise for 4 to 6 weeks except in a poorly vented area which became filmwise after 5 weeks. Only the poorly vented area showed distinct tube darkening. Promotion by a 2-month-old thio silane solution was of somewhat poorer quality although still 90% dropwise after 6 weeks.